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Electron neutrino opacity in magnetised media

Esteban Roulet
Depto. de Física Teórica, Universidad de Valencia,
E-46100, Burjasot, Valencia, Spain
E-mail: roulet@goya.ific.uv.es

Abstract: We study the effects of strong magnetic fields ($B \gtrsim 10^{13}$ G) in the cross section for $\nu_e n \to pe$ scattering in the presence of a degenerate electron background. This can be relevant for the ν_e propagation in the proto–neutron star stage after supernovae collapse. We find that for field strengths $B \gtrsim 10^{16}$ G$(E_\nu/10$ MeV$)^2$ the ν_e opacity is sizeably affected by the magnetic field and can lead to a shift in the location of the electron neutrino sphere towards lower densities. We discuss the implications that this may have for scenarios proposed to explain the observed pulsar velocities.

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There has recently been a renewed interest in the study of the neutrino propagation at very high densities ($\rho > 10^{10}\mathrm{g/cm^3}$) and in the presence of strong magnetic fields ($B > 10^{13}$ G), in connection with the proposed explanations for the observed pulsar recoil velocities, which could arise from an asymmetry in the neutrino emission during the first seconds after the collapse of massive stars [1, 2, 3, 4, 5, 6, 7, 8]. The high densities reached in the proto–neutron star stage result in the trapping of the neutrinos within the so–called neutrino spheres, i.e. the surfaces from which the optical depth becomes of order unity. The location of these surfaces depends on the neutrino flavour because of the effects of the charged current interactions, which allow the ν_e to escape only from larger radii (mainly because of the large $\nu_e n \to pe$ cross section) than the $\nu_{\mu,\tau}$ (and their antineutrinos), while the $\bar{\nu}_e$–sphere radius is in between the previous ones (due mainly to the $\bar{\nu}_e p \to ne^+$ interaction).

The fact that the radii of these neutrino spheres is flavour dependent is at the basis of the mechanism suggested by Kusenko and Segrè [6] to account for the pulsar kicks, which is based on the resonant conversion $\nu_e \to \nu_\tau$ just in the region between those two spheres, so that trapped ν_es oscillating to ν_τs become free to escape. As was noticed by those authors, the large magnetic fields which may be present in the proto–neutron star (as evidenced by the large B fields, $B \sim 10^{12}$–2×10^{13} G, observed in young pulsars), may be responsible for a distortion of the resonance density for the MSW conversion and can lead then to an asymmetry in the average energy of the emitted ν_τs. For this mechanism to be successful, field strengths $B \gtrsim 10^{14}$ G would be required, and some scenarios have been proposed in which such large fields may be achievable at the epoch of the supernova collapse [9, 10, 11, 12], with the maximum conceivable strength being $B \sim 10^{18}$ G, a value beyond which the magnetic energy becomes larger than the gravitational one.

We want here to study whether these large magnetic fields may actually affect the $\nu_e n \to pe$ cross section which determines the ν_e opacity itself and hence shift the location of the ν_e–sphere. This can happen due to the modification of the phase space distribution of the final state electrons in the presence of the magnetic field. It has indeed been shown that the neutron decay and electron capture processes [13, 14, 15] are affected significantly for $B \gtrsim B_c \equiv m_e^2/e = 4.41 \times 10^{13}$ G, in which case the spacing between the first Landau levels becomes larger than the electron mass m_e. For larger B fields, the separation between Landau levels may become comparable to the typical energies of the neutrinos emitted in supernovae, $E_{\nu_e} \sim 1$–30 MeV, and hence the neutrino opacity may also be changed.

The reaction rates for the $\nu_e n$ interaction in the presence of magnetic fields were considered in [16], focussing in the conditions for β equilibrium. Furthermore, the possible B dependence of the $\nu_e n \to pe$ cross section has been recently invoked as an explanation of the pulsar kicks if an asymmetry in the magnetic field distribution develops during the first second after the collapse [17]. A detailed evaluation of the magnetic field effects on the ν_e opacity is however lacking and will be the subject of the present work.

We will then concentrate in the study of the ν_e produced during the Kelvin Helmholtz cooling phase [18, 19], just after the time when the shock produced by the core bounce dissociates the heavy elements initially present and make the ν_e opacity in the shock heated region to be dominated by the reaction $\nu_e n \to pe$ (before that, scattering off nuclei dominates).

At zero density and in the absence of magnetic fields, the total cross section for this reaction is (see e.g. [20])

$$\sigma_0 = \frac{G^2}{\pi}(g_V^2 + 3g_A^2)(E_\nu + Q)^2\sqrt{1 - m_e^2/(E_\nu + Q)^2}, \tag{1}$$

where $G \equiv G_F \cos\theta_c$ stands for the product of the Fermi coupling and the Cabibbo mixing while $g_V = 1$ and $g_A = 1.26$ are the vector and axial nucleon couplings. Since the energy transfered to the recoiling proton is $\mathcal{O}(E_\nu^2/m_p)$, which is negligible for the neutrino energies we are interested in, one has $E_e \simeq E_\nu + Q$, with $Q \equiv m_n - m_p = 1.293$ MeV. This 'elastic' approximation is indeed quite reliable [21] for densities below the nuclear saturation one and for non–degenerate neutrinos ($\mu_\nu \simeq 0$), as is the case of interest here for the study of the location of the neutrino–sphere.

At finite densities (but still with $B = 0$), the main modification will come from an overall electron blocking factor $1 - f_e(E_e)$ multiplying the cross section, where

$$f_e(E) = \frac{1}{1 + \exp\left[(E - \mu)/T\right]}, \tag{2}$$

is the Fermi Dirac distribution at temperature T, with μ being the electron's chemical potential.

Considering now a non–vanishing magnetic field, the matrix element for the process will remain essentially unaffected (we average over initial spins neglecting the neutron polarisation, which is small for $B < 10^{17}$ G) [14, 15]. The main modification will then come from the available phase space for the electrons, since the phase space factor for $B = 0$

$$\sum_e = 2\int \frac{d^3p}{(2\pi)^3} \tag{3}$$

has now to be replaced with

$$\sum_e = \frac{eB}{(2\pi)^2}\sum_{n=0}^{n_{max}} g_n \int dp_z, \tag{4}$$

where $g_0 = 1$ and $g_n = 2$ for $n \geq 1$ are the degeneracies of the Landau levels of energy

$$E_n = \sqrt{p_z^2 + m_e^2(1 + 2nB_*)}, \tag{5}$$

and we have introduced $B_* \equiv B/B_c$.

Computing the $\nu_e n \to pe$ cross section in the presence of the magnetic field we then obtain

$$\begin{aligned}\sigma &= \tfrac{G^2}{4\pi}(g_V^2 + 3g_A^2)m_e^2 B_* \textstyle\sum_n g_n \int dp_z \delta(E_\nu + Q - E_e)\left[1 - f_e(E_e)\right] \\ &= \tfrac{G^2}{2\pi}(g_V^2 + 3g_A^2)m_e^2 B_* \left[1 - f_e(E_\nu + Q)\right](E_\nu + Q)\textstyle\sum_n g_n/p_z^{(n)},\end{aligned} \tag{6}$$

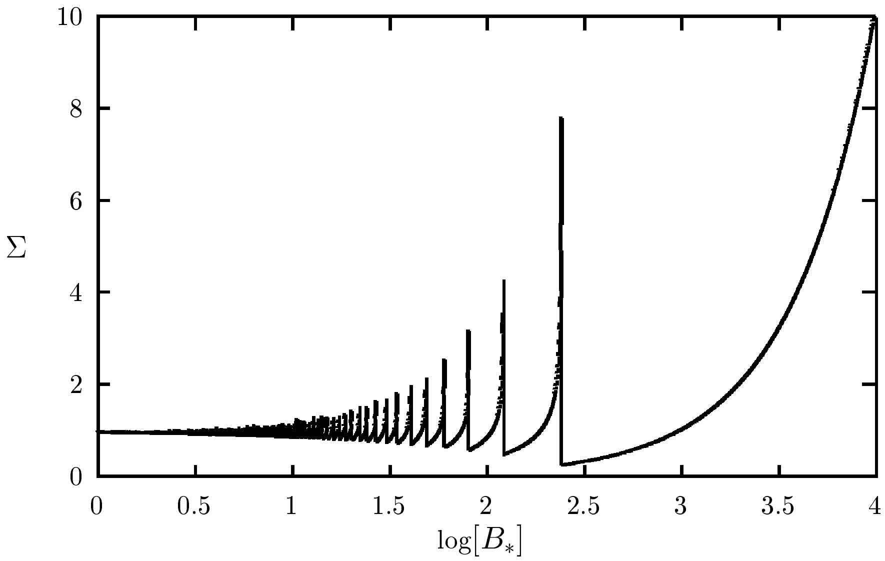

Figure 1: Normalised cross section, $\Sigma \equiv \sigma/\sigma_0$, for $E_\nu = 10$ MeV, as a function of the magnetic field $B_* \equiv B/B_c$, in the absence of background matter and for $T = 0$.

where $p_z^{(n)} \equiv \sqrt{(E_\nu + Q)^2 - m_e^2(1 + 2nB_*)}$. The maximum Landau level accessible to the final state electron, for a given energy of the initial neutrino, is

$$n_{max} = \mathrm{int}\left\{\frac{1}{2B_*}\left[\left(\frac{E_\nu + Q}{m_e}\right)^2 - 1\right]\right\}. \tag{7}$$

In figure 1 we plot the cross section in vacuum, normalised to the $B = 0$ one ($\Sigma \equiv \sigma/\sigma_0$), as a function of the magnetic field, fixing $E_\nu = 10$ MeV for definiteness.

For field values

$$B > \frac{1}{2}\left[\left(\frac{E_\nu + Q}{m_e}\right)^2 - 1\right] B_c \simeq 10^{16}\mathrm{G}\left(\frac{E_\nu}{10\ \mathrm{MeV}}\right)^2, \tag{8}$$

only the lowest Landau level ($n = 0$) contributes to the phase space, and in this case one has

$$\sigma \simeq \frac{G^2}{2\pi}(g_V^2 + 3g_A^2) m_e^2 B_*, \tag{9}$$

which grows linearly with B and is independent of the neutrino energy. For smaller magnetic field values, $n_{max} \geq 1$ and hence more Landau levels contribute to the sum in eq. (6). The singular behaviour present each time that a new Landau level opens up, which is similar to the one found in the β–spectrum in n–decay [14], arises from the p_z^{-1} factor from $dp_z = E/p_z dE$, and is expected to be somewhat smeared once the proton recoil momentum is included and its effects also averaged out once a distribution of neutrino energies is considered. In the limit of small magnetic fields, n_{max} will be large and we may approximate the sum over Landau levels as

$$\sum_{n=0}^{n_{max}} \frac{g_n}{p_z^{(n)}} = \quad (E_e^2 - m_e^2)^{-1/2} + 2\sum_{n=1}^{n_{max}}(E_e^2 - m_e^2(1 + 2nB_*))^{-1/2}$$

$$\begin{aligned} &\simeq && (E_e^2 - m_e^2)^{-1/2}\left[1 + \tfrac{x}{B_*}\int_{2B_*/x}^{1} dy(1-y)^{-1/2}\right] && (10)\\ &= && (E_e^2 - m_e^2)^{-1/2}\left[1 + \tfrac{2x}{B_*}\sqrt{1 - 2B_*/x}\right], \end{aligned}$$

where we defined $x \equiv (E_e/m_e)^2 - 1$. From this we get, for $B_* \ll x$,

$$\Sigma \simeq 1 - \frac{B_*}{2x}, \tag{11}$$

and the $B = 0$ result in eq. (1) is then asymptotically recovered.

In the presence of background matter, the electron density is given by

$$n_e = \sum_e f_e(E) = \frac{m_e^2 B_*}{(2\pi)^2} \sum_n g_n \int dp_z f_e(E_n), \tag{12}$$

and a similar expression holds for the positron density, with the replacement $\mu \to -\mu$.

Taking into account that for typical proto–neutron star temperatures a positron background is also generally present, we can introduce the electron fraction as

$$Y_e = \frac{n_e - n_{e^+}}{n_p + n_n}. \tag{13}$$

The star density may then be written as

$$\rho \simeq m_p(n_p + n_n) = \frac{m_p}{Y_e}(n_e - n_{e^+}). \tag{14}$$

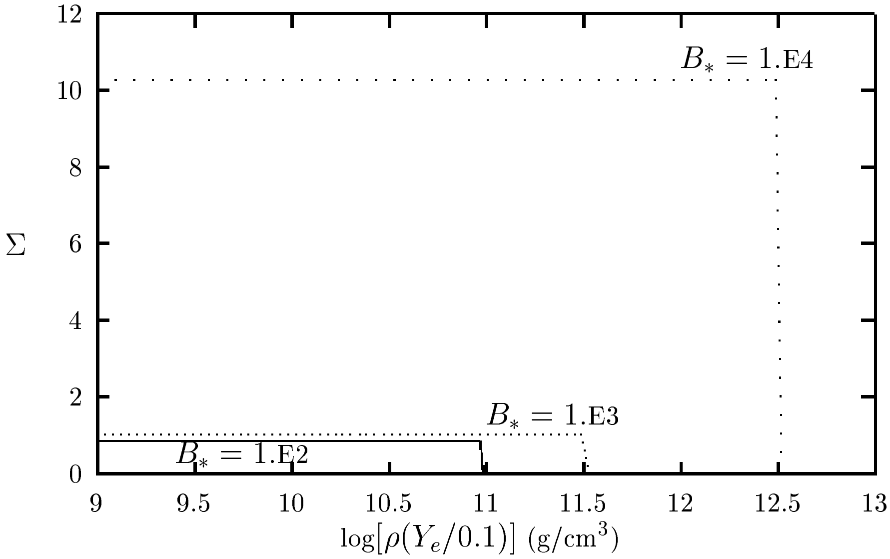

Figure 2: Normalised cross section vs. background matter density, fixing $E_\nu = 10$ MeV and for magnetic fields $B_* = 10^2$, 10^3 and 10^4. For $B_* < 10^2$ the results are almost insensitive to B.

Considering now the effects of electron degeneracy in the $\nu_e n$ scattering, we plot in figure 2 the normalised cross section as a function of the density, neglecting for the moment temperature effects ($T \ll E_\nu, \mu$), for different values of B. Under this approximation $f_e(E) = \theta(\mu - E_\nu - Q)$, and hence the effect of the background is just to block the final

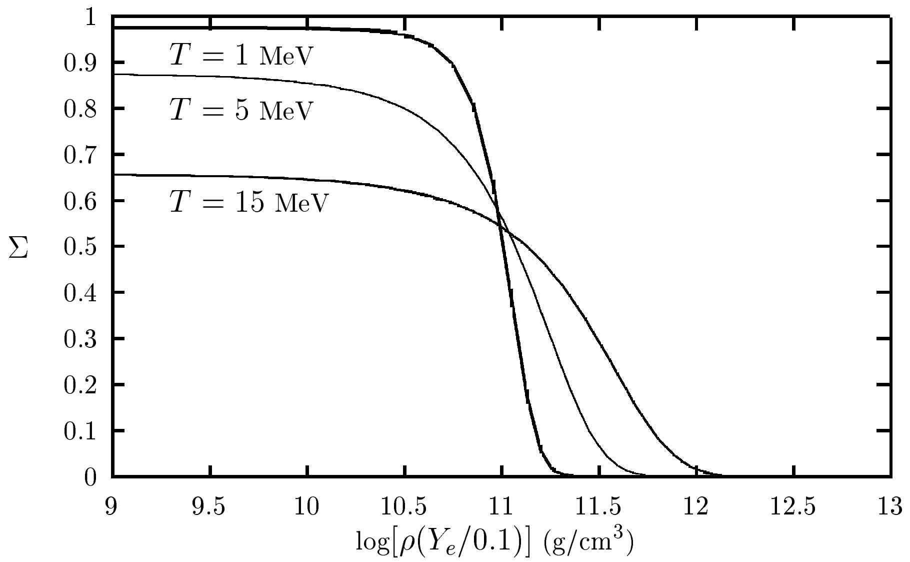

Figure 3: Temperature dependence of the Σ vs. ρ relation, fixing $E_\nu = 10$ MeV and $B_* = 1$.

state electrons, resulting in a maximum density ρ_{max} beyond which no scattering can take place and σ vanishes. For densities larger than ρ_{max} only neutral currents will contribute to the neutrino opacity.[1] Increasing the size of B will modify the size of σ as discussed in relation with figure 1, and, due to the B dependence of the electron density, also change the maximum density beyond which σ vanishes, i.e. $\rho_{max} = \rho_{max}(B, E_\nu)$. One has to keep in mind that large magnetic fields will also affect the Y_e values corresponding to β equilibrium and hence the details of the proto–neutron star evolution.

Turning now to the effects of finite temperatures, we show in figure 3 the modification of the previous picture with the inclusion of thermal distributions, plotting Σ vs. ρ for different temperatures. The main result is that some of the final states with $E_e > \mu$ become occupied at finite temperatures, leading to a partial Pauli blocking of the final state electrons, while for $E_e < \mu$, some electron states below the Fermi energy become free at non–zero temperature and the available phase space does not vanish abruptly but instead diminishes smoothly for increasing densities. It is clear from figure 3 that only the low energy part of the neutrino spectrum is affected by thermal effects, which just open up some phase space for the final state electrons and result in a non–vanishing contribution to the opacity up to much larger densities.

Let us now discuss the implications of the previous results for the emission of neutrinos. During the cooling phase the neutrinos will diffuse out from the inner regions of the proto–neutron star[2] and will be emitted from the surface of the neutrino sphere. The presence of large magnetic fields can affect the electron neutrino opacity and hence shift the radius of the neutrino sphere. A change in this radius affects the neutrino luminosity in two

[1] Here it is useful to recall that for $B = 0$, $T = 0$, one has $\mu \simeq (3\pi^2 n_e)^{1/3} \simeq 11$ MeV$(\rho Y_e/10^{10}\mathrm{g/cm}^3)^{1/3}$, and hence in this case $\rho_{max} Y_e \simeq 10^{10}\mathrm{g/cm}^3((E_\nu + Q)/11\ \mathrm{MeV})^3$.

[2] Actually, it is the lepton number which diffuses, since neutrinos can be captured by neutrons and then reemited by electron captures, as well as being thermally pair produced.

ways: i) it modifies the area of the emitting surface ($L_\nu \propto R^2$) and ii) the temperature of the matter in the emission region can be different (the temperature associated to the neutrinos is usually obtained from a fit to the resulting neutrino spectrum, but the low energy neutrinos come from deeper layers of the star, while those of higher energies are trapped up to larger radii, making the neutrino spectrum not exactly a Fermi Dirac one).

Usually a mean neutrino sphere radius is obtained by averaging over the neutrino energy distribution, but we prefer instead to work here with an energy dependent neutrino sphere radius, since for instance when considering the Kusenko and Segrè mechanism one needs to actually compare this radius with the location of the resonance for neutrino conversion, which is energy dependent.

An estimate of the matter density at the ν_e–sphere radius R_e can be obtained from the condition of having unit optical depth, $\tau(R_e) = 1$, with

$$\tau(r) \equiv \int_r^\infty dr \ \sum_i n_i \sigma(\nu_e i \to X) \simeq \sigma \int_r^\infty dr \ n_n(r), \tag{15}$$

and where we used that the sum over possible scatterers i is dominated by the scattering $\nu_e n \to pe$.

Hence, we can write the approximate relation

$$1 = \sigma \int_{R_e}^\infty dr n_n(r) \simeq \sigma n_n(R_e) h_n, \tag{16}$$

where $h_n \equiv |d\ln n_n/dr|_{R_e}^{-1}$ is the scale height of the neutron distribution at the ν_e–sphere radius (h_n is actually not really a constant, since the density profile is not exponential). From this one can estimate the density at the neutrino–sphere as

$$\rho(R_e) \simeq \frac{m_p}{Y_n \sigma h_n} \simeq 5 \times 10^{11} \frac{\mathrm{g}}{\mathrm{cm}^3} \left(\frac{10 \ \mathrm{MeV}}{E_\nu}\right)^2 \left(\frac{1 \ \mathrm{km}}{h_n}\right) \frac{\sigma_0}{\sigma}, \tag{17}$$

where $Y_n = 1 - Y_e$ is the neutron fraction ($Y_e \sim 10^{-1}$ and $h_n \sim 10$ km typically).

Hence, a change in σ caused by a large magnetic field will shift the density of the ν_e–sphere. In particular, for very large fields $B_* \gtrsim 240(E_\nu/10 \ \mathrm{MeV})^2$, for which only the lowest Landau level contributes to σ, one has $\Sigma = (B_*/2)(m_e/E)^2$, and hence the density of the neutrino sphere will behave as B_*^{-1} for large fields.

Our results differ significantly from those used in ref. [17], where the estimate $\Sigma \simeq 0.77 B_*$ (for $B_* \gg 1$) was adopted from a simple analogy with the n–decay results [13]. As a consequence, to induce a sizeable recoil velocity from the effects on the neutrino emission resulting from an asymmetric magnetic field distribution would require much larger fields.

Let us also notice that if $\rho(R_e)$ is bigger than the ρ_{max} discussed in connection with figure 2 (this can happen for small E_ν), the $\nu_e n \to pe$ cross section will vanish, neglecting temperature effects, inside the neutrino sphere. In this case, only magnetic fields large enough to increase ρ_{max} beyond $\rho(R_e)$ may be able to affect sizeably the ν_e opacity.

Considering now the scenario proposed by Kusenko and Segrè [6], we note that a value of $\Sigma > 1$ would shift the ν_e–sphere to larger radii, and hence lower densities. This

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may be interesting in order to allow the resonant conversion $\nu_e \to \nu_\tau$ to happen inside the ν_e–sphere but with smaller values of Δm^2 (since $\Delta m^2 \propto N_e$), possibly within the cosmologically acceptable values corresponding to $\sum m_{\nu_i} < 92(\Omega_\nu h^2)$ eV. This could cure one of the main drawbacks of the model, which is the need for large neutrino masses, with $\Delta m^2 > (100\ \text{eV})^2$. However, for typical neutrino energies, $E_\nu > 5$ MeV, to increase Σ significantly would require quite large fields, $B > 10^{16}$ G, and hence this possibility seems also difficult to implement.

Other mechanisms based on large magnetic fields to produce the pulsar velocities, such as by means of the B dependence of the differential cross section of URCA processes [1, 2] or of νe scattering [3], do not depend directly on the precise location of the neutrino spheres, so that the process discussed here will not interfere with those scenarios.

In conclusion, we have studied the behaviour of the $\nu_e n \to pe$ cross section in very strong magnetic fields, finding that it can lead to sizeable modifications of the neutrino opacities in proto–neutron stars for $B \gtrsim 10^{16}\ \text{G}(E_\nu/10\ \text{MeV})^2$. The impact of this for scenarios proposed to explain the observed pulsar velocities seems then marginal in view of the extremely high fields required.

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